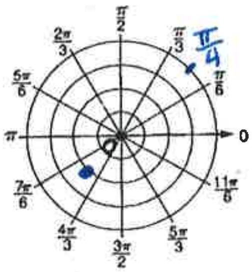
Ch.8 Review (part 2) NO CALCULATOR!! NAME: Key

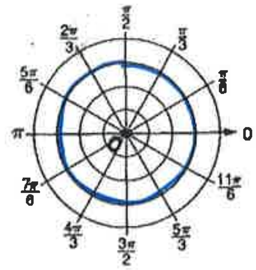
Graph each of the following.

1. $\left(-2, \frac{\pi}{4}\right)$

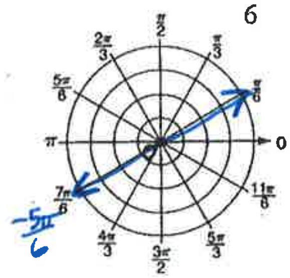


2. $\left(2, -\frac{2\pi}{3}\right)$

3. $r = 3$



4. $\theta = \frac{-5\pi}{6}$



Find each product, quotient, or power and express the result in rectangular form. ***Let*** $z_1 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ ***and*** $z_2 = 0.5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(*) polar form (*) rectangular form

5. $z_1 z_2$ mod $4(.5) = \boxed{2}$ amp $\frac{2\pi}{3} + \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$ $\boxed{2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)} = 2\left(\frac{-\sqrt{3}}{2} + \frac{1}{2}i\right) = \boxed{-\sqrt{3} + i}$

6. $\frac{z_1}{z_2}$ $\frac{4}{.5} = \boxed{8}$ $\frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{6}$ or $\boxed{\frac{\pi}{2}}$ $\boxed{8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)} = 8(0 + 1i) = \boxed{8i}$

7. z_1^2 $4(4) = \boxed{16}$ $\frac{2\pi}{3} + \frac{2\pi}{3} = \boxed{\frac{4\pi}{3}}$ $\boxed{16\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)} = 16\left(-\frac{1}{2} + -\frac{\sqrt{3}}{2}i\right) = \boxed{-8 - 8\sqrt{3}i}$

Find each power or root. Express the result in rectangular form. Use DeMoivre's Theorem.

8. $(\sqrt{2} + \sqrt{2}i)^4 = n$

$(\sqrt{2}, \sqrt{2})$ Quad I

$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$ $r = \sqrt{4}$ $r = 2$

$\tan\theta = \frac{\sqrt{2}}{\sqrt{2}}$ $\tan\theta = 1$ $\theta = \frac{\pi}{4}$

$= 2^4\left(\cos 4\left(\frac{\pi}{4}\right) + i\sin 4\left(\frac{\pi}{4}\right)\right)$

$= 16(\cos\pi + i\sin\pi) = 16(-1 + 0i)$

$= \boxed{-16}$

9. $\sqrt[3]{64i}$ $y_3 = n$ Hint: rewrite in the form (a+bi) and use a fractional exponent.

$(0 + 64i)^{1/3} = n$

$(0, 64)$ on boundary

$r = \sqrt{0^2 + 64^2}$ $r = \sqrt{64^2}$ $r = 64$

$\tan\theta = \frac{64}{0}$ $\tan\theta = $ undef $\theta = \frac{\pi}{2}$

$= 64^{1/3}\left[\cos\frac{1}{3}\left(\frac{\pi}{2}\right) + i\sin\frac{1}{3}\left(\frac{\pi}{2}\right)\right]$

$= 4\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$

$= 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= \boxed{2\sqrt{3} + 2i}$

10. Find the polar coordinates of the point with rectangular coordinates $(4, -4\sqrt{3})$. Use $0 \le \theta < 2\pi$ and $r \ge 0$.

$r = \sqrt{4^2 + (-4\sqrt{3})^2} = \sqrt{16 + 48} = \sqrt{64} = 8$

$\tan\theta = \frac{-4\sqrt{3}}{4}$ $\tan\theta = -\sqrt{3}$ $\boxed{\theta = \frac{5\pi}{3} \text{ Quad IV}}$

$\boxed{\left(8, \frac{5\pi}{3}\right)}$

11. Find the rectangular coordinates of the point with polar coordinates $\left(6, \frac{7\pi}{4}\right)$.

$x = r\cos\theta$ $x = 6\left(\cos\frac{7\pi}{4}\right)$ $x = 6\left(\frac{\sqrt{2}}{2}\right)$ $x = 3\sqrt{2}$

$y = 6\left(\sin\frac{7\pi}{4}\right)$ $y = 6\left(-\frac{\sqrt{2}}{2}\right)$ $y = -3\sqrt{2}$

$\boxed{(3\sqrt{2}, -3\sqrt{2})}$

12. Write the polar equation $r = 5$ in rectangular form.

$\sqrt{x^2 + y^2} = 5$ $\boxed{x^2 + y^2 = 25}$

Simplify.

13. $2(3 + i) - (4 - i)$

$= 6 + 2i - 4 + i$

$= \boxed{2 + 3i}$

14. $(2 - 4i)(3 + 5i)$

$6 - 12i + 10i - 20i^2$

$6 - 2i + 20$

$= \boxed{26 - 2i}$

15. $\frac{4 - 3i}{5 + 2i} \cdot \frac{(5 - 2i)}{(5 - 2i)}$

$= \frac{20 - 15i - 8i + 6i^2}{25 - 4i^2} = \frac{20 - 23i - 6}{25 + 4} = \frac{14 - 23i}{29} = \boxed{\frac{14}{29} - \frac{23}{29}i}$

CHECK ANSWERS:

~~$-4\sqrt{2} + 4\sqrt{2}i$~~

~~$2\sqrt{3} + 2i$~~

~~$\frac{14}{29} - \frac{23}{29}i$~~

~~$-8 - 8\sqrt{3}i$~~

~~$-\sqrt{3} + i$~~

~~$26 - 2i$~~

~~$2 + 3i$~~

~~$(3\sqrt{2}, -3\sqrt{2})$~~

~~-16~~ ~~2~~ ~~8~~ ~~16~~

~~$8i$~~ ~~$\frac{\pi}{2}$~~ ~~$\frac{4\pi}{3}$~~ ~~$\frac{5\pi}{6}$~~

~~$x^2 + y^2 = 25$~~

~~$\left(8, \frac{5\pi}{3}\right)$~~

~~$2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$~~

~~$4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$~~

16. Express $2\sqrt{3} - 2i$ in polar form. Quad IV

$r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$ $= \sqrt{12 + 4}$ $= \sqrt{16}$ $r = 4$

$\tan\theta = \frac{-2}{2\sqrt{3}}$ $\tan\theta = \frac{-1}{\sqrt{3}}$ $\theta = \frac{11\pi}{6}$

$= \boxed{4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$

17. Express $8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in rectangular form.

$= 8\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \boxed{-4\sqrt{2} + 4\sqrt{2}i}$